# Circuit Power Considered Energy Efficiency In Decode-and-Forward Relaying

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***Abstract*—In this paper we analyze Energy Efficiency (EE) and Spectral Efficiency (SE) in relay based systems. Our system model is based on two data transmitters as sources and one relay node with Decode-and-Forward (DF) scheme. The system can work in two strategies, DF relaying and direct transmission. In addition to transmission power we consider circuit power of nodes in our Power Consumption Model (PCM) and find the best EE-SE for each strategy by optimizing transmission time and power of each node. Simulation results show that SE-EE trade off in DF relaying has better performance than direct transmission. Despite traditional research that shows exponential decreasing of EE with increase in SE, we will show by considering circuit power, EE-SE relation has a cup shape. The best EE will be achieved by optimizing power and time duration of transmission.**

## I. INTRODUCTION

Green communication has become very important and attractive for wireless system architects in recent years. One of the most important trade-offs of this topic is SE versus EE trade-off [1]. Spectral efficiency is the system throughput per unit of bandwidth while EE is defined as transmitted information bits per unit of energy. In [1] it is shown that if just the transmission energy is considered in definition of EE, it will be a monotonic decreasing function of SE. But if the device or circuit energy is also considered in definition of EE, the relation of EE and SE will change to a cup shape. In other words, by having a more practical view to consider the circuit power of each node in SE-EE trade–off, we could define a point where SE is none zero and EE is maximized.

Relay is known as a device that helps to coverage of network and also provides cooperative diversity. Cooperative diversity brings the benefits of MIMO without use of multiple antennas in the transmitter or receiver [2]. One of the open issues in SE-EE trade-off is, whether relaying can be helpful or not [3]. In [4], EE of Amplify-and-Forward (AF) relaying has been studied. In this paper DF relaying is analyzed to find which strategy (DF relaying or direct transmission) is more energy efficient. Let's consider circuit energy of nodes for both transmitter and receiver and also circuit energy in idle mode. Circuit energy of nodes in transmitting and receiving mode are equal and much higher than idle mode. Assume that the system is delay constraint that means it should transmit $B$ bits in a block duration of T. It may use a shorter duration for transmission (and reception) and for rest of the block duration, the system stays in idle mode to start a new block. The goal is to minimize the energy consumption of system. It seems to be helpful if transmission time is decreased and idle mode is increased. However, one should note that by reducing the transmission time the amount of required power to transmit $B$ bits increases and it does not help the EE of system. So there should be an optimum time for transmission (and reception).

The rest of this paper is organized as follows. In section II and III in addition to introducing system model, Energy Consumption (EC) of each strategy will be defined. In section IV we will optimize EEs of each strategy. In section V we will show results of simulations and conclusion of this paper will be presented in section VI.

## II. SYSTEM MODEL

Let's consider a system with three nodes, node A, B as transceiver and node R as Relay. Node A sends data to node B and node B sends data to node A. Node R is a half-duplex relay node that helps to transfer data between A and B in a DF cooperation scheme. So during each block of T, first node A sends bits as a packet of data to node B with the assistance of node R , then node B sends $B$ data bits to node A through relay R.

The channels between nodes are assumed to be flat fading and noise power in each node is $N_0$. Channel State Information (CSI) is perfect and assumed to be available in each node. System has to transfer $B$ bits in each direction ($A \to B$ and $B \to A$) during $T$ seconds. But it does not have to use all the block duration. In other words, if transmission time is $T_t$, the system transmits in $2T_t$ Seconds and stays in idle mode (to start new block) in $T - 2T_t$ Seconds. Therefore each node has three modes: transmission mode, reception mode and idle mode.

As mentioned above for more practical constraints one needs to consider circuit power of device in the power consumption model. Hence, there are two kinds of power:

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- Transmission power, $P^T$ (which is well known and could be obtained by Shannon formula).
- Circuit power, $P^c$.

According to [5] the circuit power consumption is mainly due to RF power circuit and it is independent of bit rate. So circuit powers are constant. In each mode there is a unique circuit power $P^C$. Also it is reasonable to assume that the circuit power of transmission and receive modes are identical and much larger than that of idle mode ($P^{ct} = P^{cr} > P^{ci}$). Also channel gain of $A \to B$ , $A \to R$ and $R \to B$ are indicated respectively by, $h_{ab}$, $h_{ar}$, $h_{br}$.

## III. ENERGY CONSUMPTION MODEL (ECM)

In this section for two strategies of direct transmission and DF relaying ECM will be introduced. Then in next section these ECMs will be optimized for maximum EE.

### A. Direct Transmission

In direct transmission there is no relay. First, node A transmits *B* bits to node B and then node B transmits *B* bits to node A. Each node sends its data in $T_D$ seconds. System is delay constraint and has *T* seconds for transmitting *2B* data bits in both directions. But it can fulfill transmitting process in $2T_D$ seconds and stay idle in the remaining time ($T - 2T_D$).

When system is transmitting data in one direction transmitter consumes $P^T$ Watts for transmission and $P^{ct}$ Watts for circuit power, while receiver just requires $P^{cr}$ Watts for its circuit power. When system is in idle mode both nodes spend $P^{ci}$ Watts for the circuit power.

So the ECM for direct transmission is obtained as:

$$E_D = T_D(\frac{P_a^T}{\varepsilon} + P_a^{ct} + P_b^{cr}) + T_D(\frac{P_b^T}{\varepsilon} + P_b^{ct} + P_a^{cr}) + (T - 2T_D)(P_a^{ci} + P_b^{ci}) \quad (1)$$

Where $\varepsilon \in (0,1]$ indicates the power amplifier efficiency.

Define $P_D^{c1} = P_a^{ct} + P_b^{cr}$, $P_D^{c2} = P_b^{ct} + P_a^{cr}$, $P_D^{ci} = P_a^{ci} + P_b^{ci}$.

The ECM can be obtained as:

$$E_D = T_D(\frac{P_a^T}{\varepsilon} + P_D^{c1} - P_D^{ci}) + T_D(\frac{P_b^T}{\varepsilon} + P_D^{c2} - P_D^{ci}) + TP_D^{ci} \quad (2)$$

### B. DF Relaying

In DF relaying assume a two-hop relay with node R that is located in middle of distance between node A and node B. It works in half-duplex mode and cooperation scheme is decode-and-forward.

So each node requires two hopes in its transmission time $T_O$. First, node A sends its data to relay (duration time for first hop is $T_O/2$ seconds). In the second hop relay sends these bits with a different coding to node B, duration time for the second hop is also $T_O/2$. Then node B starts to send its message for node A in two hops (again each hop has $T_O/2$ second duration). It should be clear that when one node is in first hop of its transmission, relay is in receiving mode and destination node is in idle mode.

Then in the second hop of transmission, source node is in idle mode, relay is in transmitting mode and destination is in receiving mode. The ECM in one-way relaying could be obtained as follow:

$$\begin{aligned} E_O &= \tfrac{T_O}{2}(\frac{P_a^T}{\varepsilon} + P_a^{ct} + P_r^{cr} + P_b^{ci} + \frac{P_r^T}{\varepsilon} + P_r^{ct} + P_b^{cr} + P_a^{ci}) \\ &+ \tfrac{T_O}{2}(\frac{P_b^T}{\varepsilon} + P_b^{ct} + P_r^{cr} + P_a^{ci} + \frac{P_r^T}{\varepsilon} + P_r^{ct} + P_a^{cr} + P_b^{ci}) \\ &+ (T - 2T_O)(P_a^{ci} + P_b^{ci} + P_r^{ci}) \end{aligned}$$

$$E_O = T_O(\frac{P_a^T + P_r^T}{2\varepsilon} + P_O^{c1} - P_O^{ci}) + T_O(\frac{P_b^T + P_r^T}{2\varepsilon} + P_O^{c2} - P_O^{ci}) + TP_O^{ci} \quad (3)$$

Where:

$$P_O^{c1} \equiv \frac{P_a^{ct} + P_r^{cr} + P_b^{ci} + P_r^{ct} + P_b^{cr} + P_a^{ci}}{2}$$

$$P_O^{c2} \equiv \frac{P_b^{ct} + P_r^{cr} + P_a^{ci} + P_r^{ct} + P_a^{cr} + P_b^{ci}}{2}$$

$$P_O^{ci} = P_a^{ci} + P_b^{ci} + P_r^{ci} \quad (4)$$

## IV. OPTIMIZATION OF ENERGY EFFICIENCY

In this section EE optimization of each strategy will be discussed. First let's define EE. Energy efficiency is known as the capacity of a system per unit of power. In other words, one can say energy efficiency in a system is the number of bits transmitted per unit of energy. In our system *2B* bits are transmitted in block duration of *T* and since the energy consumption of each strategy is obtained in the above, hence, energy efficiency could be obtained as:

$$\eta_{EE} = \frac{2B}{E} \quad (5)$$

where *E* is energy consumption per block of each strategy.

Maximizing $\eta_{EE}$ for a given *B* is equivalent to minimizing E in each strategy. In the above it was shown that E in each strategy depends on node powers. It is clear that for a given bandwidth and a given number of bits, the required power and time for transmission are inversely proportional. By optimizing transmission time and power of nodes one can minimize E in each strategy, to maximize EE for a given number of bits. This will be the main approach of the proposed method as follows.

### A. Direct Transmission

To minimize E of direct transmission consider the following optimization problem:

$$\min_{T_t, P_a^T, P_b^T} \quad T_D(\frac{P_a^T}{\varepsilon} + P_D^{c1} - P_D^{ci}) + T_D(\frac{P_b^T}{\varepsilon} + P_D^{c2} - P_D^{ci}) + TP_D^{ci}$$

$$s.t \quad 2T_D \le T \;, P_a^T \le P_{max}^t, P_b^T \le P_{max}^t \qquad (6)$$

Where $P_{max}^t$ is the maximum available power in each node. It is constant and is the same for all nodes.

Assume direct transmission where the system bandwidth is *W,* and *B* bits are transmitted in $T_D$ seconds in each direction. According to the Shannon relation capacity of system in each direction is obtained by:

$$\frac{B}{T_D} = W \log(1 + \frac{P_a^T |h_{ab}|^2}{N_0}) \qquad (7)$$

$$\frac{B}{T_D} = W \log(1 + \frac{P_b^T |h_{ab}|^2}{N_0}) \qquad (8)$$

In this strategy transmission time and transmission power have simple relations and one can define the optimization problem on any of them. So the joint optimization problem (6) changes to:

$$\min_{T_D} \quad T_D(\frac{N_0(2^{\frac{B}{WT_D}} - 1)}{\varepsilon |h_{ab}|^2} + P_D^{c1} - P_D^{ci})$$

$$+ T_D(\frac{N_0(2^{\frac{B}{WT_D}} - 1)}{\varepsilon |h_{ab}|^2} + P_D^{c2} - P_D^{ci}) + TP_D^{ci}$$

$$s.t \quad 2T_D \le T \;, T_D > T_{D\min} \qquad (9)$$

where $T_{D\min}$ is defined as minimum required time for transmitter to send data when it consumes its maximum power, and

$$T_{D\min} = \frac{B}{W log_2(1 + \frac{P_{\max}^T |h_{ab}|^2}{N_0})} \qquad (10)$$

The objective function in (9) is a convex function of $T_D$ and all of the constraints are also convex. So the problem leads to convex optimization. Now by taking derivative of objective function in (9) with respect to $T_D$ and setting it to zero one can find the optimum $T_D$ and minimize $E_D$. According to the definition of the $P_D^{c1}$ and $P_D^{c2}$ we know that they are equal and we have:

$$\frac{dE_D}{dt} = \frac{d}{dt}(T_D(\frac{N_0(2^{\frac{B}{WT_D}} - 1)}{\varepsilon |h_{ab}|^2} + P_D^{c1} - P_D^{ci})$$

$$+ T_D(\frac{N_0(2^{\frac{B}{WT_D}} - 1)}{\varepsilon |h_{ab}|^2} + P_D^{c2} - P_D^{ci}) + TP_D^{ci}) = 0$$

$$[\frac{N_0(2^{\frac{2B}{WT_D}} - 1)}{\varepsilon |h_{ab}|^2} + P_D^{c1} - P_D^{ci}] - \frac{N_0 \ln 2}{\varepsilon |h_{ab}|^2} 2^{\frac{2B}{WT_D}} \frac{2B}{WT_D} = 0 \qquad (11)$$

Now the optimum $T_D$ can be found, but it does not have a closed form and a recursive method should be used to find it from (11).

If the optimum $T_D$, is denoted by $T_{Dopt}$, then :

$$[\frac{N_0(2^{\frac{2B}{WT_{Dopt}}} - 1)}{\varepsilon |h_{ab}|^2} + P_D^{c1} - P_D^{ci}] = [\frac{N_0(2^{\frac{2B}{WT_{Dopt}}} - 1)}{\varepsilon |h_{ab}|^2} + P_D^{c2} - P_D^{ci}]$$

$$= \frac{N_0 \ln 2}{\varepsilon |h_{ab}|^2} 2^{\frac{2B}{WT_{Dopt}}} \frac{2B}{WT_{Dopt}} \qquad (12)$$

and optimum energy efficiency is obtained as:

$$\eta_{EEopt}^D = \frac{2B}{\frac{2BN_0(\ln 2)}{\varepsilon |h_{ab}|^2 W} 2^{\eta_{SEopt}^D} + TP_D^{ci}} \qquad (13)$$

Where $\eta_{SEopt}^D \equiv \frac{2B}{WT_{Dopt}}$.

*B. DF Relaying*

According to (3) the optimization problem for one relay scheme is defined as:

$$\min_{T_O, P_a^T, P_b^T, P_r^T} \quad T_O(\frac{P_a^T + P_r^T}{2\varepsilon} + P_O^{c1} - P_O^{ci})$$

$$+ T_O(\frac{P_b^T + P_r^T}{2\varepsilon} + P_O^{c2} - P_O^{ci}) + TP_O^{ci}$$

$$s.t \quad 2T_O \le T \;, P_a^T \le P_{max}^t, P_b^T \le P_{max}^t, P_r^T \le P_{max}^t$$

(14)

In this case the joint optimization problem of EC should be modified to a simpler optimization problem that has just time as a variable. Therefore transmission power should be expressed as a function of transmission time. For this propose, first transmission power is minimized and then it is derived as a function of time. However, since the cooperative scheme is decode-and- forward, the capacity (for one direction for example when node A is transmitting) is derived as [6]:

$$C = \frac{B}{T_O} = \frac{W}{2} \min\{\log_2(1 + \frac{|h_{ar}|^2 P_a^T}{N_0}), \log_2(1 + \frac{|h_{br}|^2 P_r^T}{N_0})\}$$

(15)

Similar to [4] we don't consider direct link. Power transmission minimization problem is defined as:

$$\min_{P_a^T, P_r^T} \quad P_a^T + P_r^T$$

$$s.t \qquad P_a^T \le P_{max}^t, P_r^T \le P_{max}^t, (15) \tag{16}$$

This optimization problem can be solved in two different cases.

*1) if* $|h_{ar}|^2 P_a^T > |h_{br}|^2 P_r^T$ *:*

In this case we can write $P_r^T$ as a function of transmission time:

$$P_r^T = \frac{(2^{\frac{2B}{T_O W}} - 1)N_0}{|h_{br}|^2} \tag{17}$$

Also (16) could be changed to:

$$\min_{P_r^T} \quad P_r^T (1 + \frac{|h_{br}|^2}{|h_{ar}|^2})$$

$$s.t \qquad P_r^T \le P_{max}^t, (17) \tag{18}$$

Since the objective function is linear in (18), the solution of (16) for this case can be derived as:

$$P_{aopt}^T + P_{ropt}^T = (2^{\frac{2B}{T_O W}} - 1)N_0 (\frac{1}{|h_{br}|^2} + \frac{1}{|h_{ar}|^2}) \tag{19}$$

Similar to direct transmission $T_{O\min 1}$ is obtained as:

$$T_{O\min 1} = \frac{B}{W log_2 (1 + \frac{P_{\max}^T |h_{br}|^2}{N_0})} \tag{20}$$

*2) if* $|h_{ar}|^2 P_a^T < |h_{br}|^2 P_r^T$ *:*

Unlike the previous case, in this case $P_a^T$ determines system capacity. Where

$$P_a^T = \frac{(2^{\frac{2B}{T_O W}} - 1)N_0}{|h_{ar}|^2} \tag{21}$$

In this case is $T_{O\min 2}$ defined as:

$$T_{O\min 2} = \frac{B}{W log_2 (1 + \frac{P_{\max}^T |h_{ar}|^2}{N_0})} \tag{22}$$

And (16) can change to:

$$\min_{P_a^T} \quad P_a^T (1 + \frac{|h_{ar}|^2}{|h_{br}|^2})$$

$$s.t \qquad P_r^T \le P_{max}^t, (21) \tag{23}$$

It is interesting that solution of (23) is exactly the same as the solution of (18). So both cases have equal answers for (16) but $T_{O\min 1}$ and $T_{O\min 2}$ are different.

When node B is transmitting and node A is receiving the problem is the same and it is similar to what we was derived before.

So the joint optimization problem (14) can be simplified to a one variable optimization problem as:

$$\min_{T_O} \quad T_O (\frac{(2^{\frac{2B}{T_O W}} - 1)N_0}{2\varepsilon |h_{eff}|^2} + P_O^{c1} - P_O^{ci})$$

$$+ T_O (\frac{(2^{\frac{2B}{T_O W}} - 1)N_0}{2\varepsilon |h_{eff}|^2} + P_O^{c2} - P_O^{ci}) + TP_O^{ci}$$

$$s.t \qquad 2T_O \le T \;, T_O \ge \max\{T_{O\min 1}, T_{O\min 2}\} \tag{24}$$

Where $|h_{eff}|^2 \equiv \frac{1}{\frac{1}{|h_{br}|^2} + \frac{1}{|h_{ar}|^2}}$.

Solution of (24) is the same as solution of (9) and could be obtained as:

$$\eta_{EEopt}^O = \frac{2B}{\frac{2BN_0(\ln 2)}{\varepsilon |h_{eff}|^2 W} 2^{\eta_{SEopt}^O} + TP_O^{ci}} \tag{25}$$

Where $\eta_{SEopt}^O \equiv \frac{2B}{WT_{Oopt}}$.

## V. Simulation and Results

In this section circuit power considerations on relation between EE-SE are investigated. Also we can find out how optimal EE will change as a function of SE in each strategy and find for a given SE which strategy has the best performance (which strategy is more energy efficient).

In the simulation it is assumed that channels between nodes have Rayleigh distribution with path loss exponent $\alpha = 4$. We consider the relay position in the middle of two nodes. Other parameters are: $W = 10MHz$, $T = 5ms$, $P_{\max} = 10$ Watt, $N_0 = 4 \times 10^{-4}$ Watt, $\varepsilon = 0.35$.

In figure 1, for each node it is assumed that $P^{ct} = P^{cr} = 100$ mw. One can note that DF relaying is more energy efficient than direct transmission. The peak of DF transmission efficiency is twice that of direct transmission. In high spectral efficiency conditions, energy efficiency of both strategies converge to the same value.

In figure 2, the effect of circuit power on energy efficiency of DF relaying transmission is presented. Obviously, without circuit power consideration in ECM, energy efficiency is a monotonic decreasing function of spectral efficiency. With circuit power consideration in ECM, the EE-SE curve becomes like a cup shape and can maximize energy efficiency in a point that has non-zero spectral efficiency.

In figure 3, the effect of circuit power on energy efficiency of both strategies is shown. And obviously in both cases, with/without circuit power consideration, energy efficiency of DF relaying is better than direct transmission.

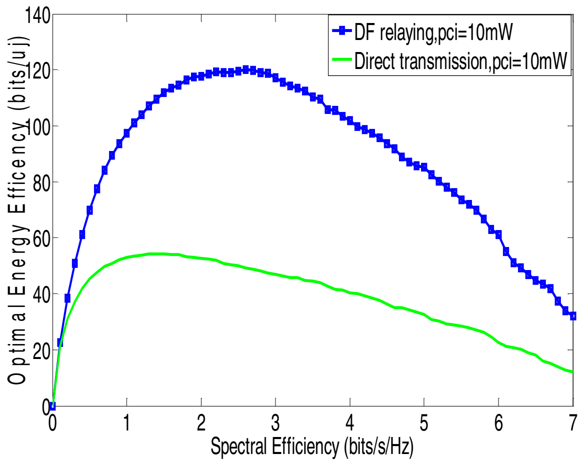


Figure 1. The comparison of EE-SE with circuit power consideration in DF relaying and direct transmission

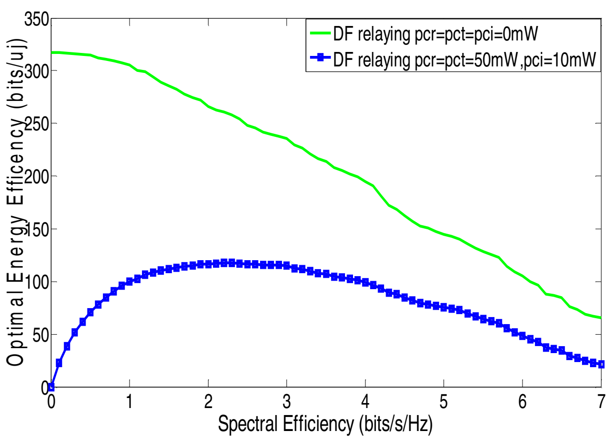


Figure 2. The comparison of EE-SE with and without circuit power consideration in DF relaying

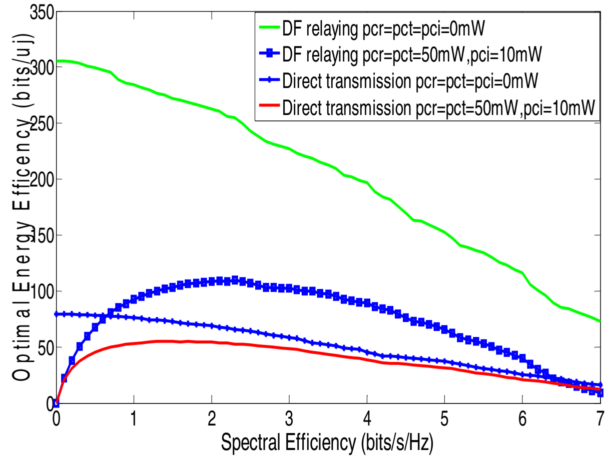


Figure 3. The comparison of EE-SE with and without circuit power for both strategies

## VI. CONCLUSION

In this paper we analyzed energy efficiency and spectral efficiency in relay based systems. The system model is based on two data transmitters as source and one relay system with Decode-and-Forward scheme. The comparison of two strategies (DF relaying and direct transmission) was considered. In addition to transmission power, circuit power of nodes in our Power Consumption Model was considered. The best EE-SE was obtained for each strategy by optimizing transmission time and each node power. Simulation results show that SE-EE trade off in DF relaying has better performance compared to direct transmission. Despite traditional research results that show exponential decreasing of EE with increase in SE, by considering circuit power, EE-SE relation has a cup shape. The best EE is achieved by optimizing transmission power and time duration.

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